

# Math 3070/6070 Homework 6

Due: No submission

1. (4.1) A random point  $(X, Y)$  is distributed uniformly on the square with vertices  $(1, 1)$ ,  $(1, -1)$ ,  $(-1, 1)$ , and  $(-1, -1)$ . That is, the joint pdf is  $f(x, y) = \frac{1}{4}$  on the square. Determine the probabilities of the following events.

1.  $X^2 + Y^2 < 1$
2.  $2X - Y > 0$
3.  $|X + Y| < 2$

2. (4.5)

1. Find  $\Pr(X > \sqrt{Y})$  if  $X$  and  $Y$  are jointly distributed with pdf

$$f(x, y) = x + y, \quad 0 \leq x \leq 1, \quad 0 \leq y \leq 1$$

2. Find  $\Pr(X^2 < Y < X)$  if  $X$  and  $Y$  are jointly distributed with pdf

$$f(x, y) = 2x, \quad 0 \leq x \leq 1, \quad 0 \leq y \leq 1$$

3. (4.11) Let  $U$  = the number of trials needed to get the first head and  $V$  = the number of trials needed to get two heads in repeated tosses of a fair coin. Are  $U$  and  $V$  independent random variables?
4. (4.17) Let  $X$  be an exponential(1) random variable, and define  $Y$  to be the integer part of  $X + 1$ , that is

$$Y = i + 1 \quad \text{if and only if} \quad i \leq X < i + 1, \quad i = 0, 1, 2, \dots$$

1. Find the distribution of  $Y$ . What well-known distribution does  $Y$  have?
  2. Find the conditional distribution of  $X - 4$  given  $Y \geq 5$ .
5. (4.20)  $X_1$  and  $X_2$  are independent  $N(0, \sigma^2)$  random variables.

1. Find the joint distribution of  $Y_1$  and  $Y_2$ , where

$$Y_1 = X_1^2 + X_2^2 \quad \text{and} \quad Y_2 = \frac{X_1}{\sqrt{Y_1}}$$

2. Show that  $Y_1$  and  $Y_2$  are independent.